Select the best answer to each question. The choice "NOTA" denotes "None of the Above".

1) The solution to the system of equations $\begin{cases} x+2y=5 \\ 2x+3y=8 \end{cases}$ is (x_1, y_1). What is the value of $x_1 y_1$?

A) -3 B) -1.5 C) 2 D) 3 E) NOTA

2) Which of the following describes the graph of $x+y=3$ in the xy-plane?

A) a line with positive slope
B) a line with negative slope
C) a pair of intersecting lines
D) a pair of parallel lines
E) NOTA

3) Let S be the solution set to $-1 \le 2x-1 < 3$ and let T be the solution set to $0 < 3x-4 \le 2$. Find, in interval notation, $(S \cap T)$.

A) $[0,2)$ B) $[0,2]$ C) $\left(\frac{4}{3},2\right)$ D) $\left(\frac{4}{3},2\right]$ E) NOTA

For questions 4-5, consider the system of equations $\begin{cases} 3x+2y=1 \\ 2x+3y=5 \end{cases}$ with solution (x_1, y_1).

Hint: For neither question do you need to actually find (x_1, y_1).

4) What is the value of $x_1 + y_1$?

A) 0.8 B) 1 C) 1.2 D) 6 E) NOTA

5) How many ordered pairs of integers (a,b) exist such that $0 < a < 100$, $0 < b < 100$, and $ax_1 + by_1 = 0$?

A) 7 B) 9 C) 11 D) 13 E) NOTA

6) Find all values of x such that $(x^2-4x-5)(x^2+5x+6) > 0$.

A) $x < -3$ or $x > 5$
B) $x < -1$ or $x > 5$
C) $x < -3$ or $-2 < x < -1$ or $x > 5$
D) all reals
E) NOTA

7) For how many positive integers $n < 100$ is $(n-5)(n-23)(n-68)$ positive?

A) 46 B) 48 C) 50 D) 52 E) NOTA

8) Find the number of distinct integer solutions to the equation $\left(x^2 - x - 1\right)^{x+2} = 1$.

A) 2 B) 3 C) 4 D) 5 E) NOTA

9) Let *a* and *b* be real numbers such that $a \le \sum_{k=0}^{n} \frac{k}{3^k} < b$ for all positive integers *n*. What is the sum of the largest possible value of *a* and the smallest possible value of *b*?

A) $3/4$ B) $5/6$ C) $13/12$ D) $4/3$ E) NOTA

10) Find the sum of the solutions to $9^x - 30 \cdot 3^x + 100 = 0$.

A) $\log_3\left(15 + 5\sqrt{5}\right)$ B) $1 + \frac{1}{\log(3)}$ C) $\frac{2}{\log(3)}$ D) 30 E) NOTA

11) Consider that the roots of the equation $x^2 + kx + 1 = 0$ are *a* and *b*. If *k* is a real number determined by rolling a fair, standard, six-sided die and letting *k* be the number rolled on the die, what is the probability that *a* and *b* are distinct real numbers?

A) $1/3$ B) $1/2$ C) $2/3$ D) $5/6$ E) NOTA

For questions 12-13, consider that $n!! = \begin{cases} n(n-2)(n-4)(n-6)\cdots(2) & \text{for even } n \\ n(n-2)(n-4)(n-6)\cdots(1) & \text{for odd } n \end{cases}$.

12) Compute $(7!!) + (8!!)$.

A) 297 B) 479 C) 489 D) 45360 E) NOTA

13) Compute $\log_2\left(\frac{2012!!}{1006!}\right)$.

A) 1005 B) 1006 C) 2004 D) 2012 E) NOTA

14) Find the number of integers x that satisfy the inequality $|x+10|>|2x+1|$.

A) 5 B) 8 C) 11 D) 12 E) NOTA

15) Find the discriminant of the polynomial $2012x^2-2012x+2012$.

A) $-3\cdot 2012^2$ B) $2012\sqrt{3}$ C) $2012i\sqrt{3}$ D) $5\cdot 2012^2$ E) NOTA

16) Find the area of the locus of points (x,y) in the xy-plane such that $|x|+|y|\le C$ where $C>0$ is a constant.

A) C^2 B) $\sqrt{2}C^2$ C) $2C^2$ D) $4C^2$ E) NOTA

17) Find the area of the locus of points (x,y) in the xy-plane such that $\max\{|x|,|y|\}\le C$ where $C>0$ is a constant. Hint: If you are unfamiliar with the notation, $\max\{a,b\}=\begin{cases} a & \text{if } a\ge b \\ b & \text{if } b>a \end{cases}$.

A) C^2 B) $\sqrt{2}C^2$ C) $2C^2$ D) $4C^2$ E) NOTA

18) Solve the equation for $x>0$: $\dfrac{1}{x}+\dfrac{1}{x+2}=1$. What is x^6?

A) 4 B) 8 C) 27 D) 64 E) NOTA

19) Two sides of a triangle have lengths 4 and 5. Given that the third side has integer length, find the sum of all possible perimeters of the triangle.

A) 98 B) 99 C) 107 D) 126 E) NOTA

20) Consider the positive integers α, β, γ, and δ such that $\begin{cases} \alpha>\beta>\gamma>\delta \\ \alpha+\beta+\gamma+\delta=2012 \\ \alpha^2-\beta^2+\gamma^2-\delta^2=2012 \end{cases}$. Find the maximum possible value of α. *Hint:* Factor the third equation using difference of squares.

A) 1003 B) 1004 C) 1005 D) 1006 E) NOTA

For questions 21-22, consider that one popular way to solve a system of linear equations is with Cramer's Rule. Consider the system of equations $\begin{cases} 2x+y+z=3 \\ 7x+3y-2z=4 \\ 3x-6y+10z=12 \end{cases}$.

21) Find the determinant of the coefficient matrix, D, for this system of equations.

A) -91 B) 36 C) 91 D) 145 E) NOTA

22) Find the determinant of the numerator matrix for the variable x, D_x, for this system of equations.

A) -70 B) -42 C) 0 D) 42 E) NOTA

For questions 23-24, consider that Mr. Snow is going to randomly select $1 \le k \le 10$ distinct numbers from the set $\{1,2,3,\ldots,10\}$.

23) Given that $k=3$, what is the probability that the smallest number that Mr. Snow selects is 1?

A) $1/10$ B) $3/10$ C) $1/3$ D) $2/5$ E) NOTA

24) Let P_k be the probability that the smallest number that Mr. Snow selects is a 2 given that he selects k numbers. What is the maximum possible value of P_k?

A) P_3 B) P_4 C) P_5 D) P_6 E) NOTA

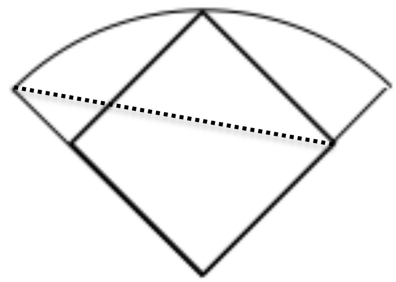
25) If $f(x-1)=x^{100}-x^{98}+x^{96}-x^{94}+x^{92}-x^{90}+\cdots+x^4-x^2+1$, find the sum of the roots of $f(x)$.

A) -100 B) 0 C) 99 D) 100 E) NOTA

26) The number of positive integers less than n that are relatively prime to n is often denoted by $\varphi(n)$. An equation for $\varphi(n)$ is given by $\varphi(n)=n\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\cdots\left(1-\frac{1}{p_k}\right)$ where $p_1,p_2,\ldots,p_k$ are the distinct prime factors of n. Find $\varphi(2012)$.

A) 502 B) 1004 C) 1006 D) 2008 E) NOTA

For questions 27-29, consider the quarter-circle shown below, which has a square inscribed in it.



27) Find the area of the quarter-circle, which is taken from the circle with equation $x^2 + y^2 = 100$.

A) 625π B) 2500π C) 5000π D) 10000π E) NOTA

28) Find the area of the square.

A) $2500\sqrt{2}$ B) 5000 C) $5000\sqrt{2}$ D) 7500 E) NOTA

29) Find the length of the dotted line in the figure (from the vertex of the square to the endpoint of the arc of the quarter-circle).

A) $50\sqrt{6}$ B) $100\sqrt{2}$ C) $100\sqrt{3}$ D) $100\sqrt{5}$ E) NOTA

30) Let $f(x)$ be a quadratic function. If $x^2 - 4x + 6 \le f(x) \le 2x^2 - 8x + 10$ for all real numbers x, and $f(12) = 182$, find $f(17)$.

A) 339.5 B) 362 C) 371.5 D) 407 E) NOTA